

Design of Spur Gear

19.04.2019

Aim: (i) To determine dimensions of gear tooth [ie. $D = mz$; $b = 4m = 10m$,
 $P_c = \pi m$; add. = m ; ded. = $1.157m$]

(ii) Module is determined by using beam strength equation of gear tooth.

(iii) Checking the dimensions of the gear tooth w.r.t. bending and wear failure (i.e. $F_{dynamic} \leq F_{beam\ strength}$ & $F_{dynamic} \leq F_{wear\ strength}$)

* Force Analysis:

For two mating or meshing gears:

(i) $\phi_1 = \phi_2 = \phi$ (Pressure angle is same)

(ii) $P_1 = P_2 = P$ (Power is same, $\eta_{mech} = 100\%$)

(iii) $F_{t1} = F_{t2} = F_t$

(iv) $m_1 = m_2 = m$.

(v) $F_{r1} = F_{r2} = F_r$.

(vi) $(F_R \text{ or } F_n) = \sqrt{F_t^2 + F_r^2} \therefore F_{R1} = F_{R2} = F_R$.

(vii) $Z_1 \neq Z_2$.

(viii) $D_1 \neq D_2$.

(ix) $N_1 \neq N_2$

(x) $T_1 \neq T_2$.

Load.	F_r	F_t
M/C component		
Gear tooth	A.C.L.	T.S.L
shaft.	T.S.L.	E.T.S.L ($e=R$) = $TSL + (TM = F_t \times R)$.

(i) Torque (T) = $\frac{60 \times 10^6 \times P}{2\pi N}$ = _____ Nmm.

(ii) $F_t = \frac{2T}{D}$

(iii) $F_r = F_t \tan \phi$.

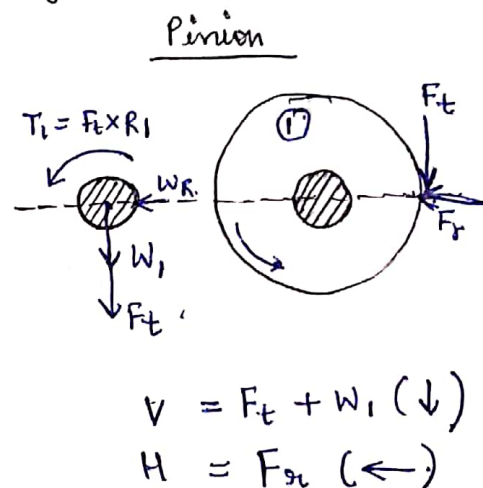
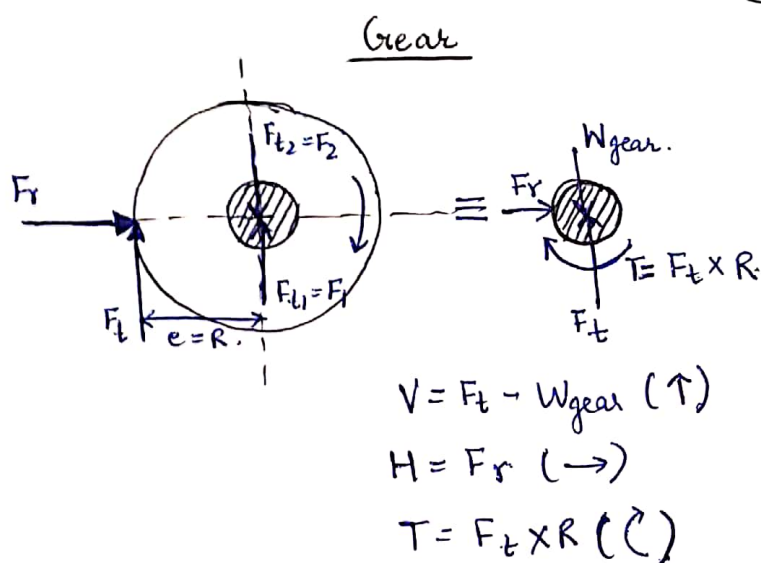
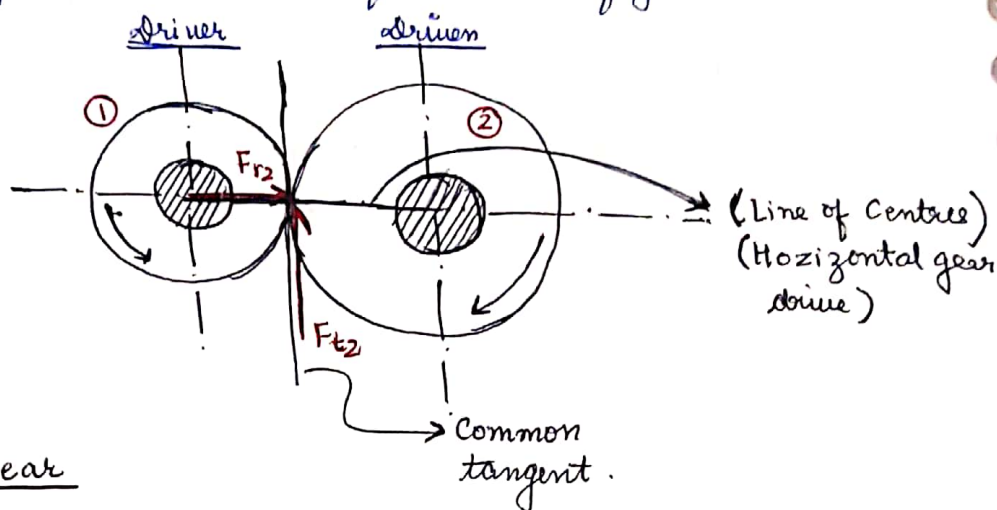
(iv) $F_R \text{ or } F_n = \sqrt{F_t^2 + F_r^2}$

* F_n will be always along the line of centre.

* F_t will be along the common tangent.

* W.r.t gear, the direction of F_r will in the direction of power transmission.

* Direction of F_t depends on direction of rotation of gear.



7 diagrams is drawn.:

• For TSL : One SFD, 1 BMD.

• For ETSL : 1 SFD, 1 BMD

• Horizontal ^{SFD} and Vertical ^{BMD} Resultant diagrams

• Torque diagram.

Out of the above 7 diagrams, SFD in both cases is not drawn as shear stress on extreme fibres are zero and BM and TM are going to be maximum.

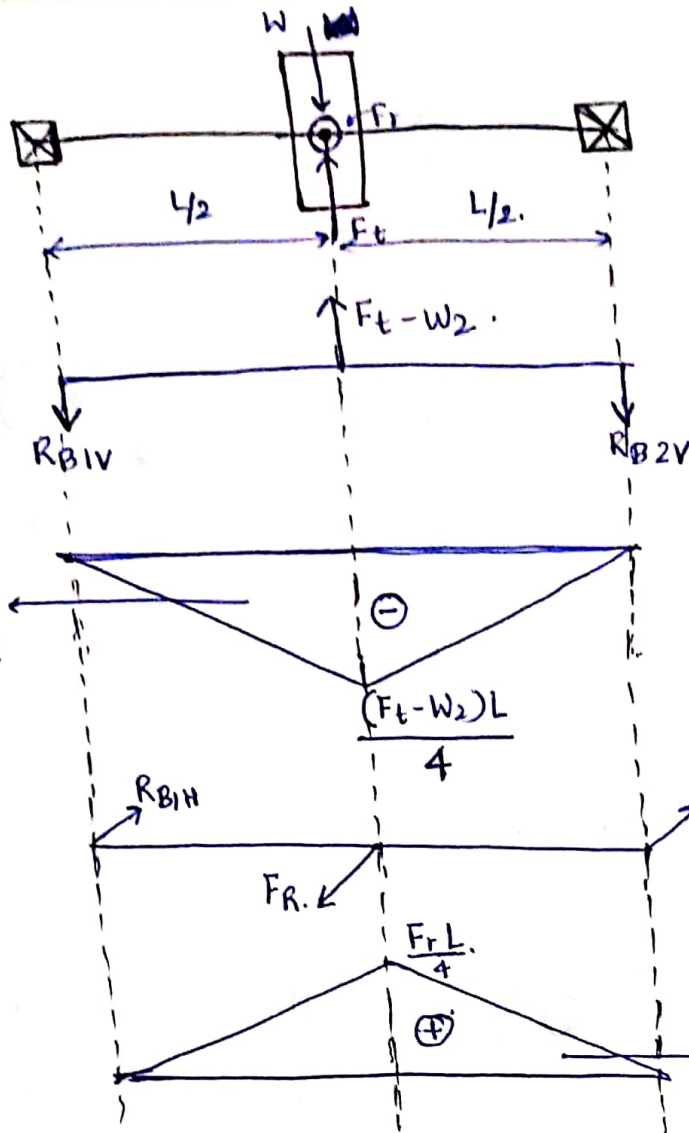
∴ Finally, there are diagrams

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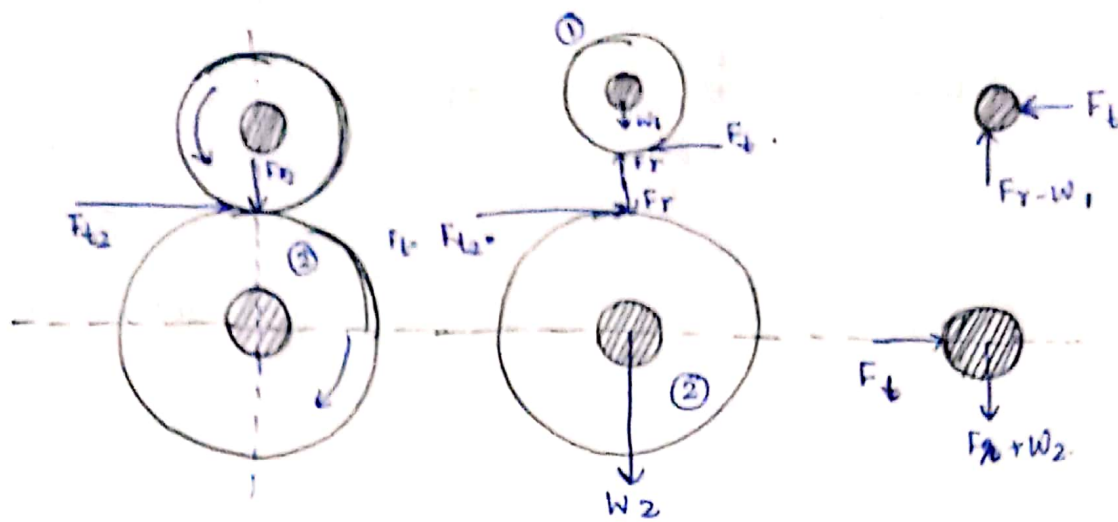
• Torque diagram.



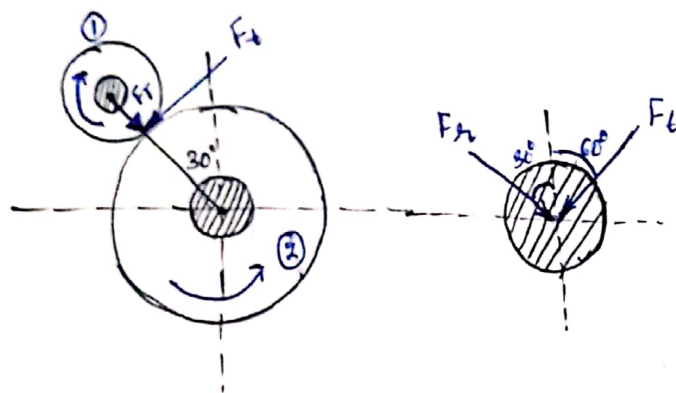
Bottom surface gets contracted

- Short and square bearings
- Gear is mounted exactly at mid span.
- Gear rotation direction depends on F_t
- Pinion rotation direction is decided by the driver shaft. It does not depend on F_t .

Back side contracts.



Vertical gear drive

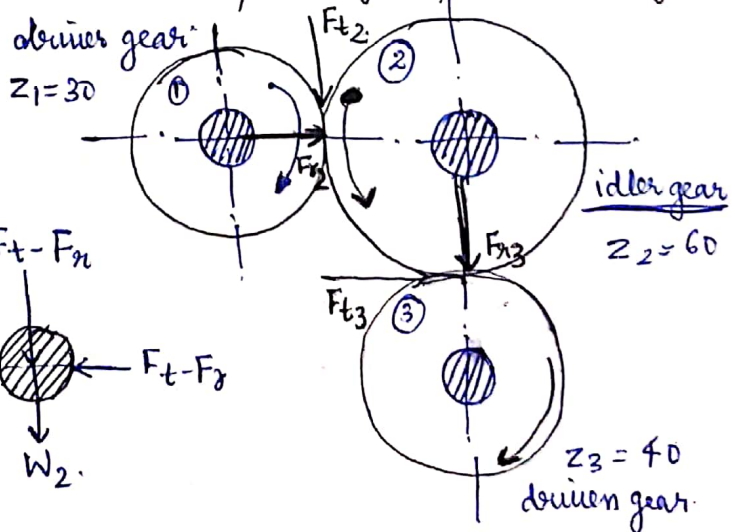
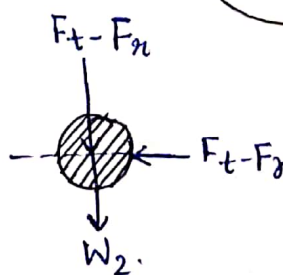
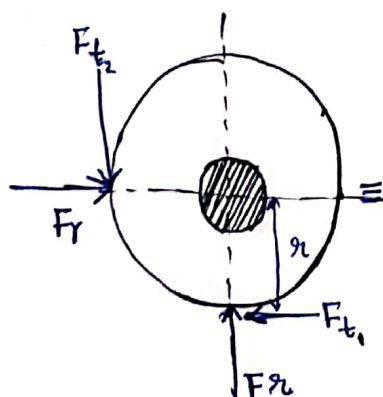


$$H = F_{t1} \sin 30^\circ - F_{t2} \sin 60^\circ$$

$$V = F_{r1} \cos 30^\circ + F_{r2} \cos 60^\circ + W_2$$

Q For the gear train as shown. Determine resultant force on idler gear shaft. Assume $P = 3.5 \text{ kW}$ at 700 rpm in CW direction; module, $m = 5 \text{ mm}$. no. of teeth on ~~pinion~~ driver gear, idler gear and driven gear are, 30, 60 and 40 respectively, pressure angle is 20° .

idler gear:



Net torque is zero $[(F_{t1} \times r_1)_{\text{CW}} + (F_{t2} \times r_2)_{\text{CW}}]$ hence, it is called idler

$$F_{t1} = F_{t2} = F_t$$

$$F_{t2} \text{ at } T \Rightarrow T_1 = F_t \times R \text{ (CW)}$$

$$F_{t1} = F_t \Rightarrow \text{HTSL} (\leftarrow)$$

$$F_{t3} = F_{t4} = F_t$$

$$F_{t3} \text{ and } F_{t4} \Rightarrow T_2 = F_t \times R \text{ (ACW)}$$

$$F_{t3} = F_t \Rightarrow \text{VTSL} (\downarrow)$$

$$\therefore T = T_1 + T_2 = 0$$

$$P_2 = T\omega = 0$$

\therefore Idler shaft is designed either ~~be~~ using bending equation or strength criterion. [Theories of failures are optional]

* For idler gear shaft:

$$H = F_t - W_1 \text{ (}\leftarrow\text{)}$$

$$V = F_t + W_2 - F_r (\downarrow) = F_t - F_r (\downarrow) \text{ neglecting weight}$$

$$R = \sqrt{H^2 + V^2} = \sqrt{2} H = \sqrt{2} V = \sqrt{2} (F_t - F_r)$$

$$\textcircled{1} T_1 = \frac{60 \times 10^6 P}{2\pi N_1} = 47746.48293 \text{ N-mm}$$

$$\textcircled{2} T_2 = 0$$

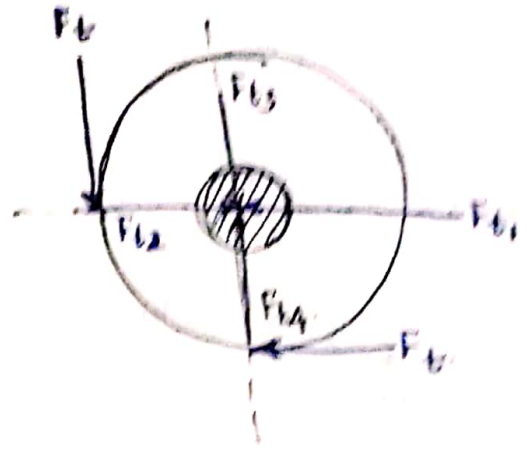
$$\textcircled{3} T_3 = \frac{60 \times 10^6 P}{2\pi N_3} \text{ or } \frac{T_3}{T_1} = \frac{Z_3}{Z_1} \text{ (for } \eta_m = 100\%) \Rightarrow T_3 = 63661.37724 \text{ N-mm}$$

$$\textcircled{4} F_{t1} = \frac{2T_1}{D_1} \text{ or } \frac{2T_3}{D_3} = 636.6198 \text{ N}$$

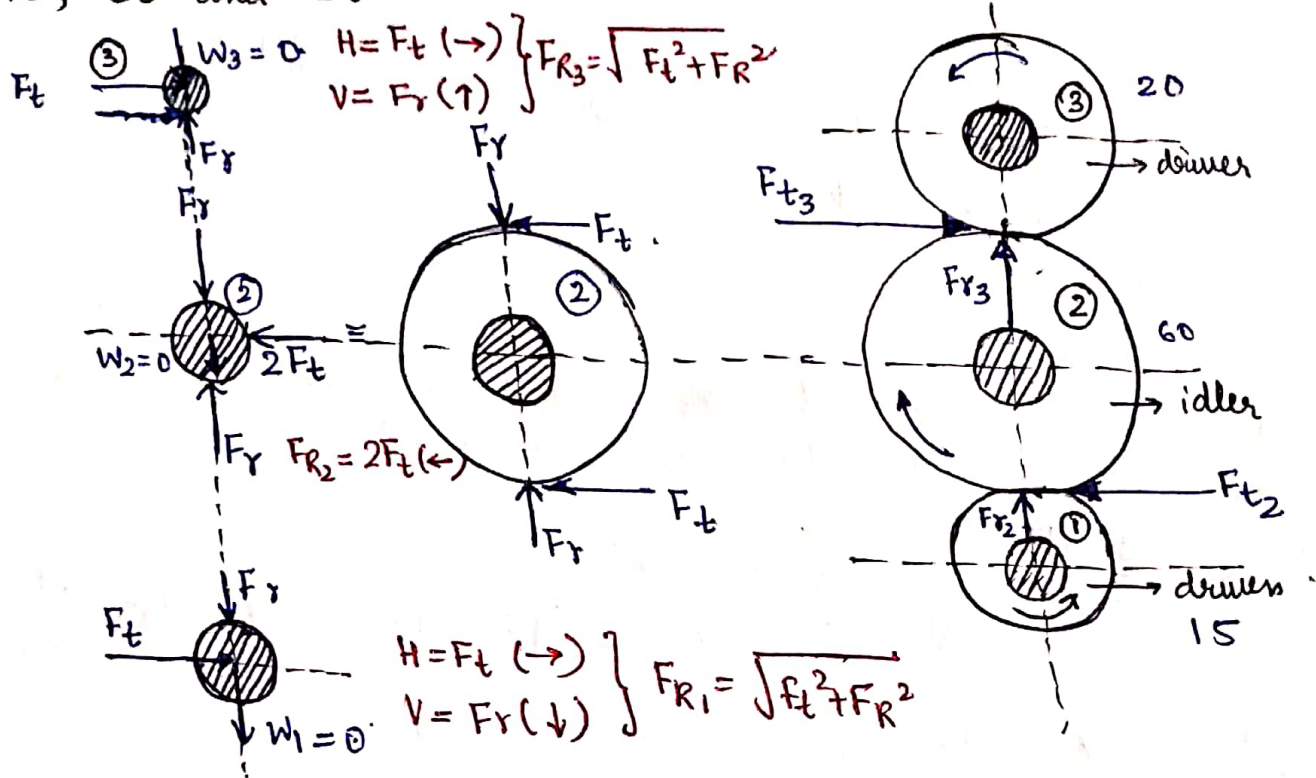
$$\textcircled{5} F_{r1} = F_{t1} \tan \phi = 231.7106 \text{ N}$$

$$\textcircled{6} (F_R)_2 = \sqrt{2} (F_{t1} - F_{r1}) = 572.628 \text{ N}$$

$$\textcircled{7} F_{R1} = F_{R3} = \sqrt{F_{t1}^2 + F_{r1}^2} = 677.47 \text{ N (by neglecting weights)}$$



- Q) For gear train as shown in Fig. determine resultant forces on driver shaft, idler gear shaft and driven shaft. $P = 10 \text{ kW}$ at 500 rpm , module = 5 mm ; $\phi = 20^\circ$; no. of teeth on driver, idler and driven are 15, 60 and 20.



$$\textcircled{1} T_1 = \frac{60 \times 10^6 \times P}{2\pi N_1} = 190985.9317 \text{ N-mm}$$

$$\textcircled{2} T_2 = 0$$

$$\textcircled{3} \frac{T_3}{T_1} = \frac{Z_3}{Z_1} = 254647.9009 \text{ N-mm}$$

$$\textcircled{4} F_t = \frac{2T_1}{D_1} \text{ or } \frac{2T_3}{D_3} = 5092.958 \text{ N}$$

$$\textcircled{5} F_r = F_t \tan \phi = 1853.685 \text{ N}$$

$$\textcircled{6} F_{R1} = F_{R3} = \sqrt{F_t^2 + F_r^2} = 5419.812 \text{ N}$$

$$\textcircled{7} F_{R2} = \sqrt{2F_t^2} = 10185.916 \text{ N}$$

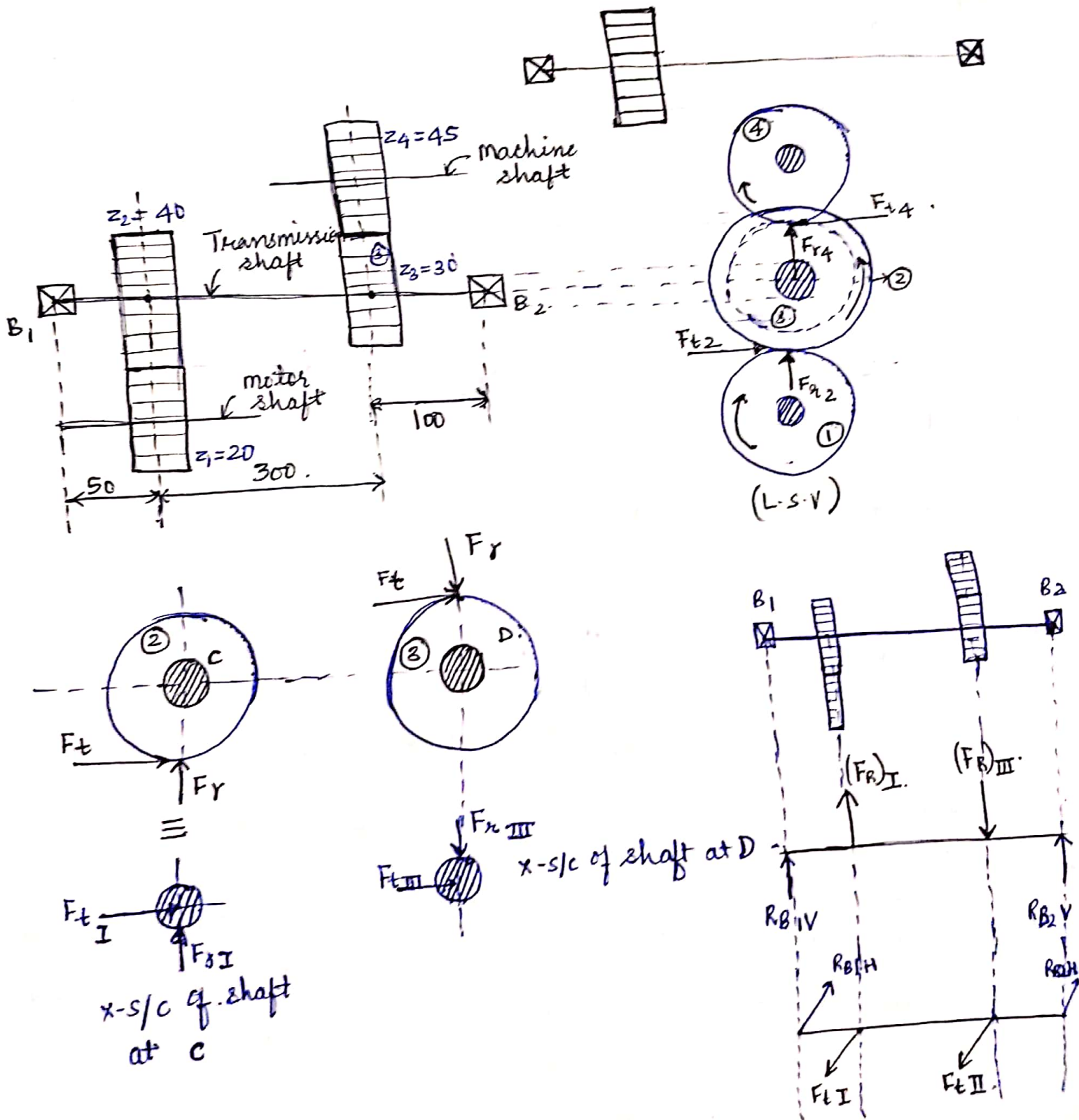
Q Determine Reactions on bearing supporting the shaft upon which gears 2 and 3 are mounted as shown in Fig. Gear 1 is driver with 20 teeth, 6mm module, 20° full depth gear, rotating at 2000 rpm in CW direction and transmitting 80 kW.

Gear 2: 40 teeth

Gear 3: 5mm module 20° F.D. with 30 teeth.

Gear 4 has 45 teeth

Solⁿ



$$T_1 = \frac{60 \times 10^6 \times f}{2\pi N_1} = 381.9718 \times 10^3 \text{ N-mm.}$$

$$(F_t)_I = \frac{2T_1}{D_1} = \frac{2T_1}{20 \times 6} = 6366.1977 \text{ N}$$

$$(F_r)_I = (F_t)_I \tan \phi = 2317.1064 \text{ N.}$$

$$\frac{T_2}{T_1} = \frac{Z_2}{Z_1} \Rightarrow T_2 = 763943.6 \text{ N-mm} = T_3 \quad \left(\begin{matrix} P_3 = P_2 \\ N_3 = N_2 \end{matrix} \right)$$

$$(F_t)_{II} = \frac{2T_3}{D_3} = 10185.9147 \text{ N.}$$

$$(F_r)_{II} = (F_t)_{II} \tan \phi_{II} = 3707.3697 \text{ N.}$$

$$F_{t1} = F_{t2} = F_{tI} \quad ; \quad F_{r1} = F_{r2} = F_{rI}$$

$$F_{t3} = F_{t4} = F_{tII} \quad F_{r3} = F_{r4} = F_{rII}$$

$$R_{B1V} = \frac{(-2317.106)(400) + (3707.36) \times 100}{450} = -1235.792 \text{ N.}$$

or $R_{B1V} = 1235.792 \text{ N} (\downarrow)$

$$R_{B2V} = +1235.792 + (-2317.106 + 3707.36) = 2626.046 \text{ N.}$$

$$R_{B1H} = \frac{(6366.1977 \times 400) + (10185.89 \times 100)}{450} = 7922.373 \text{ N} (\nearrow)$$

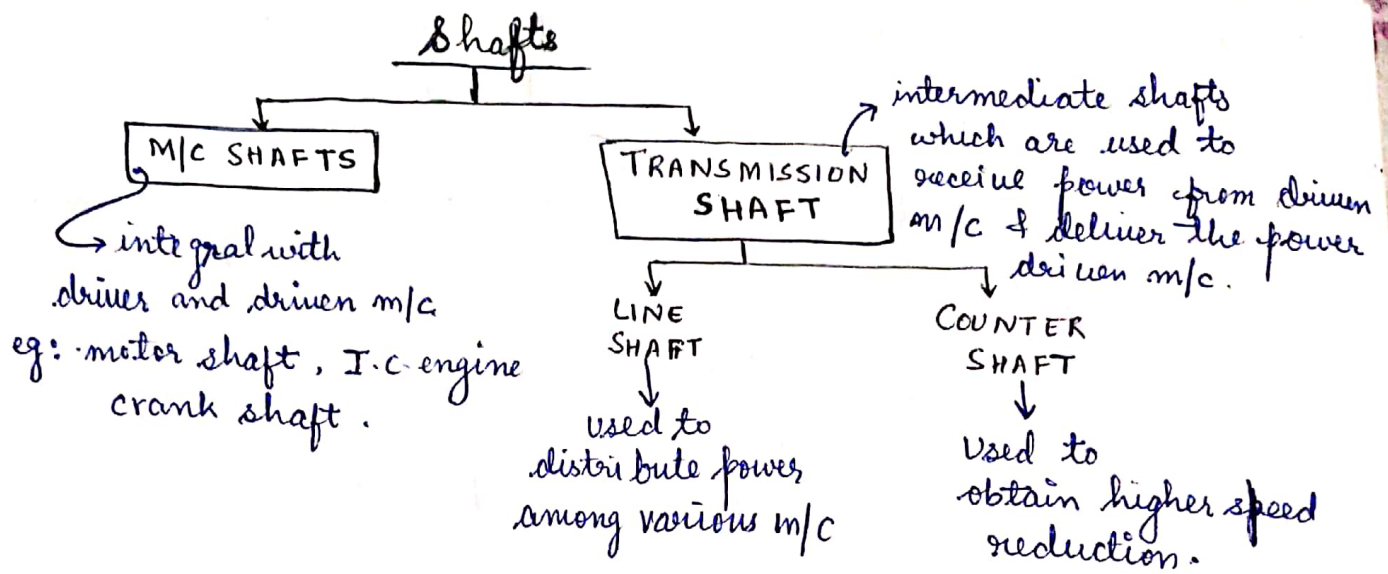
$$R_{B2H} = 8629.71 (\nearrow)$$

\therefore Resultant reaction on bearing:

$$R_{B1R} = \sqrt{(R_{B1V})^2 + (R_{B1H})^2} = 8018.176 \text{ N.}$$

$$R_{B2R} = \sqrt{(R_{B2V})^2 + (R_{B2H})^2} = 9020.4 \text{ N.}$$

- * Journal bearing is used if speed is higher and shaft works continuously
- * (Cylindrical roller) bearing ^{Anti-friction} is used if low speed and intermittent operation
- * Transmission shaft \rightarrow
 - line shaft (used when more than ^{one} machines are to operate with single motor)
 - counter shaft (here in this case, the transmission shaft is counter one)
 - used when one machine is to operate [own 2 gear drives]



* Design procedure used in spur gears :-

Input Data:

1. $P = \pi \text{ kW}$ at $\gamma \text{ rpm}$.
2. Pressure angle $(\phi) = \underline{\hspace{2cm}}$
3. Gear ratio $(G) = \underline{\hspace{2cm}}$.
4. $[\sigma_b] = \text{per bending stress} = \frac{\text{failure stress}}{N} \text{ (N=3 default)}$.
5. Lewis form factor $(y) = \underline{\hspace{2cm}}$
6. $C_v = \text{Velocity factor} = \underline{\hspace{2cm}}$.
7. $\sigma_{es} = \text{surface endurance limit} = (2.8 \times \text{BHN} - 70) \text{ MPa}$.
8. $E = \text{young's modulus}$.

Optional:

9. No. of teeth on pinion (z_1) (by using Rack & Pinion formula)
10. $\phi = b/m = 8 \text{ to } 14$ (default = 10)
11. Service factor $(C_s) = 1.2 \text{ to } 1.3$ default.

Steps: Subscripts 1 and 2 for pinion and gear respectively -

1) $T_1 = \text{torque to be transmitted by pinion}$.

(Mean torque) $T_1 = \frac{P \times 60 \times 10^6}{2\pi N_1} = \underline{\hspace{2cm}} \text{ N-mm}$.

2) $[T_1] = \text{Design torque for pinion} = T_1 \times C_s \text{ (max. torque)}$

$$3) z_1 \geq (z_1)_{\min} \text{ where } (z_1)_{\min} = \frac{2 A_w}{\sin^2 \phi} \text{ for F.D, } A_w = 1. \\ \text{for stub tooth, } A_w = 0.8$$

$$4) z_2 = G z_1$$

5) module (m):- module is determined from following eqⁿ which is obtained from beam strength of weaker gear (W.G.)

$$m \geq 1.26 \sqrt[3]{\frac{[T_1]}{([\sigma_b] Y)_{W.G.} \psi z_1}}$$

Q) ** when gear and pinion are made of same material, pinion is the weaker gear.

$$\because [\sigma_b]_1 = [\sigma_b]_2; b_1 = b_2 = b; m_1 = m_2 = m;$$

$$Y_1 < Y_2 \Rightarrow [F_s]_1 < [F_s]_2.$$

$$\text{Hence; } ([\sigma_b] Y)_{W.G.} = [\sigma_b]_1 Y.$$

** when they are made of different materials.;

$$([\sigma_b] Y)_{W.G.} = \min \text{ of } [[\sigma_b]_1 Y_1 \text{ and } [\sigma_b]_2 Y_2].$$

$$\text{Q) } Y = \pi y.$$

$$\therefore m \geq \text{_____ mm.}$$

6) dimensions of Gear train:-

$$D_1 = m z_1 = \text{_____ mm.}$$

$$D_2 = m z_2 = \text{_____ mm.}$$

$$b = \psi m = 10m = \text{_____ mm.}$$

7) F_s = beam strength of W.G.

$$F_s = ([\sigma_b] Y)_{W.G.} b m = \text{_____ N.}$$

8) F_d = dynamic load.

I Method:

$$(i) F_t = \frac{2 [T_1]}{D_1} \text{ or } \frac{2 [T_2]}{D_2}.$$

$$(ii) V_1 = V_2 = V = \frac{\pi D_1 N_1}{60} \text{ or } \frac{\pi D_2 N_2}{60} = \text{_____ m/s}$$

$$\text{iii) } C_v = \frac{3+V}{3} \text{ or } \frac{3}{3+V} \text{ (if } V \leq 10 \text{ m/s) .}$$

$$\text{(iv) } F_d = F_t \times C_v \quad \text{or } \frac{F_t}{C_v} \rightarrow C_v < 1. = \text{---} N.$$

\downarrow
 $C_v > 1$

if $F_d \leq F_s \Rightarrow$ design is safe w.r.t. bending failure.

II method: (Buckingham eqⁿ):

$$F_d = F_t + F_i \leftarrow \text{incremental load.}$$

$$F_i = \frac{21 V (C_b + F_t)}{21 V + \sqrt{C_b + F_t}}, \text{ where } C = \text{dynamic load const in N/m}$$

$$C = \frac{K e}{\left[\frac{1}{E_1} + \frac{1}{E_2} \right]} \text{ mm.}$$

e = error in tooth action in mm.

$$K = 0.11 \text{ for } \phi = 14\frac{1}{2}^\circ$$

$$K = 0.114 \text{ for } \phi = 20^\circ$$

9) Wear strength (for pinion only)

$$F_w = D_1 Q k b \text{ in N. , where } Q = \frac{2G}{G \pm 1} \quad \begin{matrix} (+) \text{ for external gears} \\ (-) \text{ for internal gears} \end{matrix}$$

* $F_d \leq F_w$

Design is safe w.r.t. wear failure.

k = material combination factor.

$$k = \frac{(\sigma_{es})^2 \sin \phi \left[\frac{1}{E_P} + \frac{1}{E_G} \right]}{1.4} \text{ MPa.}$$

$$k = 0.16 \left[\frac{BHN}{100} \right]^2 \text{ when both gear \& pinion are of STEEL}$$

Abrasive : foreign particle

corrosive : chemical action of lubricant

scoring : metal to metal contact.

pitting : due to repeated loading (fatigue.)

Q. A pair of spur gears having 20 full depth involute teeth is to transmit 20 kW. The pinion runs at 300 rpm and speed ratio 3:1. The following data are given. No. of teeth on pinion is 15. Service factor = 1; Velocity factor = $\frac{3}{3+v}$, tooth form factor $(y) = \left[0.154 - \frac{0.912}{z_p}\right]$. Face width, $b = 14\text{ m}$; allowable elastic stress for pinion and gear material are 120 MPa and 100 MPa respectively. Surface endurance limit 600 MPa. Young's modulus of pinion: 200 GPa; $E_g = 100\text{ GPa}$. Design the gear tooth and check for bending and wear failure.

- Sol^m:
- 1) $P = 20\text{ kW}$ at 300 rpm.
 - 2) Pressure ratio, $\phi = 20 \cdot \text{F.D.}$
 - 3) Gear ratio, $G = 3$.
 - 4) No. of teeth on pinion, $z_1 = 15$.
 - 5) Service factor, $C_s = 1$.
 - 6) Velocity factor, $C_v = \frac{3}{3+v}$.
 - 7) $y = \text{tooth form factor} = 0.154 - \frac{0.912}{z}$.
 - 8) face width = 14 m $\Rightarrow \psi = \frac{b}{m} = 14$.
 - 9) Permissible elastic stress for pinion $[\sigma_b]_1 = 120\text{ MPa}$.
 - 10) " " " " gear $[\sigma_b]_2 = 100\text{ MPa}$.
 - 11) Surface ^{endurance} elastic limit = $K = \sigma_{ES} = 600\text{ MPa}$.
 - 12) $E_1 = 200\text{ GPa}$; $E_2 = 100\text{ GPa}$.

Steps: Subscripts 1 and 2, represents pinion and gear respectively.

- 1) $T_1 = \text{torque to be transmitted by pinion} = \frac{P \times 60 \times 10^6}{2\pi N_1} = 636619.7724\text{ N-m}$.
- 2) design torque $[T_1] = C_s T_1 = 636619.7724\text{ N-mm}$.
for pinion.
- 3) $z_2 = \text{no. of teeth on gear} = G z_1 = 45$.
- 4) Finding the weaker gear [Finding module].

$$[\sigma_b]_1 Y_1 = 120 \times \left[0.154 - \frac{0.912}{z_1}\right] = 35.1355\text{ MPa}$$

$$[\sigma_b]_2 Y_2 = 100 \times \left[0.154 - \frac{0.912}{z_2}\right] = 42.0136\text{ MPa}$$

$\therefore [\sigma_b]_1 Y_1 < [\sigma_b]_2 Y_2 \therefore$ pinion is the weaker wheel.

$$([\sigma_b] Y)_{wg} = [\sigma_b]_1 Y_1 = 35.135\text{ MPa}$$

Hence, module should be calculated w.r.t. beam strength of pinion.

$$m \geq 1.26 \sqrt{\frac{T_1}{([\sigma_b]Y)_{W_G} \times 4 \times Z_1}} \quad , \quad m \geq 5.5676 \text{ mm} \quad \text{or} \quad \boxed{m = 6 \text{ mm}}$$

5) Dimensions of gear train

$$D_1 = mZ_1 = 90 \text{ mm}; \quad D_2 = mZ_2 = 270 \text{ mm}, \quad b = 14m = 84 \text{ mm}.$$

6) F_s = beam strength of WG

$$F_s = ([\sigma_b]Y)_{W_G} b m = 17708.292 \text{ N}.$$

7) F_d = dynamic load:

$$F_t = \frac{2[T_1]}{D_1} = 14147.10605 \text{ N}$$

$$V = \frac{\pi D_1 N_1}{60} = 1.4137 \text{ m/s}.$$

$$C_v = \frac{3}{3+V} = 0.6797.$$

$$F_d = \frac{F_t}{C_v} = 20813.7727 \text{ N}.$$

$$\therefore F_d > F_s$$

\therefore design is unsafe w.r.t. bending failure.

Hence, new module is calculated using by equating $F_d = F_s$.

8) New module:

$$F_s = F_d = 20813.7727 = ([\sigma_b]Y)_{W_G} b m = (35.1355) \times 14m \times m.$$

$$m = 6.5 \approx 7 \text{ mm}.$$

9) Dimensions of gear train (new revised)

$$D_1 = mZ_1 = 105 \text{ mm}; \quad D_2 = mZ_2 = 315 \text{ mm}, \quad b = 98 \text{ mm}.$$

$$F_s = ([\sigma_b]Y)_{W_G} b m = 24102.953 \text{ N}.$$

10) F_d = dynamic load:

$$F_t = \frac{2[T_1]}{D_1} = 12126.0909 \text{ N}.$$

$$V = \frac{\pi D_1 N_1}{60} = 1.6493 \text{ m/s}.$$

$$C_v = \frac{3}{3+V} = 0.6452$$

$$\therefore F_d = \frac{F_t}{C_v} = 18792.7575 \text{ N}$$

$\therefore F_d < F_s$ revised \therefore design is safe w.r.t. bending ^{failure} strength

ii) Wear strength: [calculated for pinion only, bcoz pinion is weaker w.r.t. wear failure $\because N_1 > N_2$]
 $F_w = D_1 Q K b$

$$Q = \frac{2G}{G_1 + 1} = \frac{3}{2}$$

$$K = \frac{(\sigma_{es})^2}{1.4} \sin \phi \left[\frac{1}{E_1} + \frac{1}{E_2} \right]$$

$$= 1.31922 \text{ MPa}$$

$$F_w = 105 \times \frac{3}{2} \times k \times 38 = 20362.16923 \text{ N}$$

$\because F_d < F_w \therefore$ design is safe w.r.t. wear failure.

* Expression for module [m]:

$$[F_s]_{w.G_1} = ([\sigma_b] Y)_{w.G_1} \cdot b \cdot m$$

$$(F_t)_{\max} \leq (F_s)_{w.G_1}$$

$$\frac{2[T_1]}{D_1} \leq ([\sigma_b] Y)_{w.G_1} \psi (m) (m) \quad \because b = 4m$$

$$\frac{2[T_1]}{m Z_1} \leq ([\sigma_b] Y)_{w.G_1} \psi (m) (m)$$

$$m^3 \geq \frac{2[T_1]}{([\sigma_b] Y) \psi Z_1}$$

$$m \geq 1.26 \sqrt[3]{\frac{[T_1]}{([\sigma_b] Y) \psi Z_1}}$$

• Det. of power transmitted by spur gear when dimensions are known :
 • I/p data: m, z, ϕ, y, ψ or $b, [\sigma_b] = \text{--- Mpa}, C_v, C_s$ and $N \text{ rpm}$

1) $F_s = ([\sigma_b] Y)_{\text{wgt}} b m$.

$([\sigma_b] Y)_{\text{wgt}} = \min. \text{ of } ([\sigma_b]_1 Y_1 \text{ and } [\sigma_b]_2 Y_2).$

2) $F_d \leq F_s$.

$\left(\frac{F_t}{C_v} \right) \text{ or } (F_t \times C_v) = F_s$ get $F_t = \text{--- kN}$

3) Power (P) = $F_t \times V = \text{--- W}$

4) Rated power = $P \times C_s = \text{--- Watts}$.

5) $F_w = D_1 \& k b = \text{---}$

$F_w \geq F_d$

\therefore dim^{ns} are safe w.r.t. wear failure.

Q Following data is given for spur gear :

(ii) Service factor = 1.2.

① No. of teeth on pinion, $z_1 = 30$.

② " " " " gear, $z_2 = 60$

③ Speed of pinion, $N_1 = 1440 \text{ rpm}$.

④ Pressure angle, $\phi = 20^\circ \text{ F.D.}$

⑤ Module, $m = 3 \text{ mm}$.

⑥ Face width, $b = 33 \text{ mm}$.

⑦ Velocity factor, $C_v = \frac{3}{3+v}$

⑧ Lewis form factor, $y = 0.154 - \frac{0.912}{z}$.

⑨ BHN = 200. both are made of steel with $S_{ut} = 560 \text{ Mpa}$.

⑩ Young's modulus, $E = 200 \text{ GPa}$.

Find: rated power on the basis of bending failure if $FOS = 1.5$ and wear strength.

solⁿ $[\sigma_b] \text{ per bending stress} = \frac{560}{1.5} = \frac{1120}{3}$

\therefore Both gear and pinion are made of same material
 \therefore pinion will be weaker wheel.

$F_s = \pi \frac{1120}{3} \left[0.154 - \frac{0.912}{30} \right] \times 33 \times 3 = 14351.59949 \text{ N}$.

$$\frac{2 \times 458366.2361}{16 \times 20.6874 \times 10} \leq m^3$$

$$\therefore m \geq 6.52 \approx \boxed{m=7}$$

⑤ Dimensions of the gear train:

$$D_1 = mZ_1 = 112 \text{ mm} ; \quad D_2 = mZ_2 = 336 \text{ mm} ; \quad b = 10m = 70 \text{ mm.}$$

⑥ F_s = beam strength of weaker gear:

$$F_s = ([\sigma_b] Y)_{wG} b \cdot m = 10136.826 \text{ N} \quad \text{--- ①}$$

⑦ F_d = dynamic load:

$$F_t = \frac{2 [T_1]}{D_1} = 8185.1113 \text{ N}$$

$$\text{Pitch line velocity } V_1 = V_2 = V = \frac{\pi D_1 N_1}{60} = 1.7593 \text{ m/s}$$

$$\text{Velocity Factor, } C_v = \frac{4.5}{4.5 + V} = 0.7189.$$

$$\therefore F_d = \frac{F_t}{C_v} = 11385.1113 \text{ N.}$$

$\therefore F_d > F_s$ \therefore design fails w.r.t. to the beam strength of gear.

⑧ Revising the module: (by equating ~~F_d~~ $F_d = F_s$).

$$\text{Using: } F_s = ([\sigma_b] Y)_{wG} \times 10 m^2 \quad (\text{eq ①})$$

$$11385.1113 = 20.6874 \times 10 \times m^2$$

$$m = 7.42$$

$$\boxed{m \approx 8}$$

⑨ Revising the dimensions of the gear train:

$$D_1 = mZ_1 = 128 \text{ mm} ; \quad D_2 = 384 \text{ mm} ; \quad b = 10m = 80 \text{ mm.}$$

⑩ Revised beam strength of weaker gear

$$F_s = 20.6874 \times 80 \times 8 = 13239.936 \text{ N.}$$

⑪ Revised dynamic load:

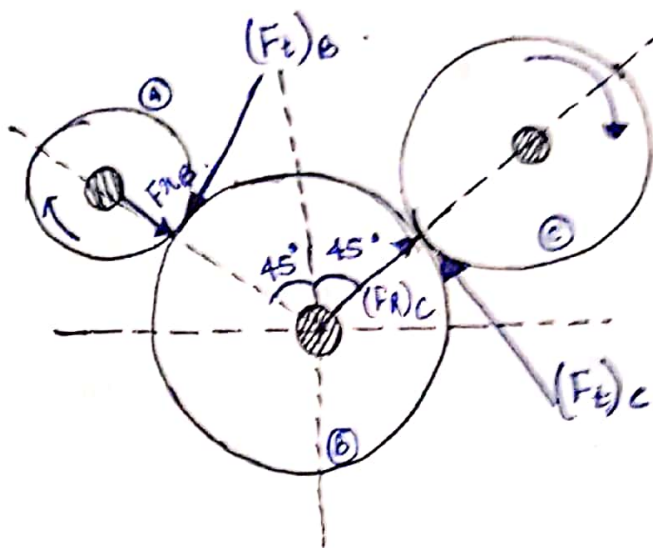
$$F_t = 7161.9724 \text{ N}$$

$$V_1 = \frac{\pi D_1 N_1}{60} = 2.0106 \text{ m/s.}$$

$$\text{Velocity factor; } C_v = \frac{4.5}{4.5 + V} = 0.6912.$$

$$F_d = \frac{F_t}{C_v} = 10361.9724 \text{ N.}$$

$\therefore F_d < F_s$: design is safe.



$$V = 2 (F_t + F_r) \cos 45^\circ$$

$$H = 0$$

$$(F_R)_2 = (\sqrt{2}) (F_t + F_r)$$

For Gear A.

$$T_A = \frac{60 \times 10^6 \times P}{2\pi N_A} = 47746.48293 \text{ N-mm}$$

$$F_t = \frac{2T_A}{D_A} = 636.6198 \text{ N}$$

$$F_r = F_t \tan \phi = 231.7166 \text{ N}$$

For Gear B

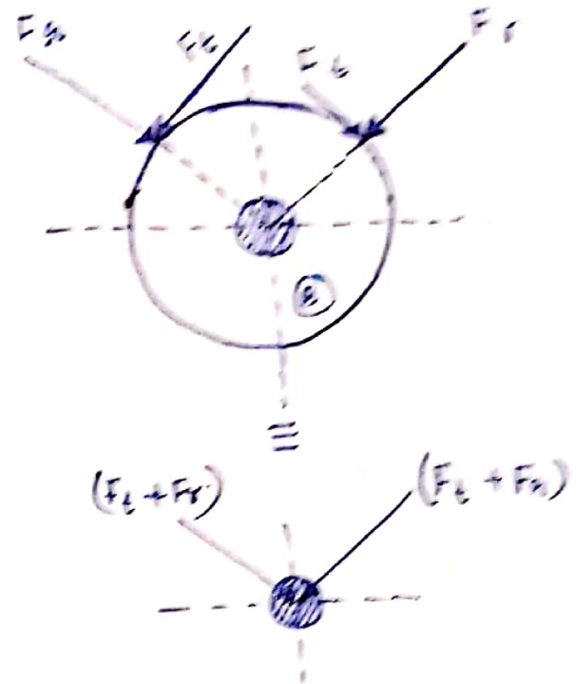
$$T_B = 0$$

For Gear C

$$\frac{T_A}{T_C} = \frac{Z_A}{Z_C} \Rightarrow T_A = 63661.97724 \text{ N-mm}$$

Now, Resultant force on idler gear shaft:

$$(F_R)_B = \sqrt{2} (F_t + F_r) = 1228 \text{ N}$$



Q1 Given:

- ① Pressure angle, $\phi = 14\frac{1}{2}^\circ$
 - ② Power transmitted, $P = 12 \text{ kW}$
 - ③ Speed of pinion, $N_1 = 3000 \text{ rpm}$
 - ④ Gear ratio $G = 3$
 - ⑤ Permissible elastic stress for pinion $[\sigma_b]_1 = 105 \text{ MPa}$
 - ⑥ " " " " gear $[\sigma_b]_2 = 60 \text{ MPa}$
 - ⑦ Surface endurance limit, $\sigma_{es} = 600 \text{ MPa}$
 - ⑧ No. of teeth on pinion, $z_1 = 16$
 - ⑨ Velocity factor, $C_v = \frac{4.5}{4.5 + V}$
 - ⑩ Form factor, $y = 0.124 - \frac{0.684}{z}$
 - ⑪ Young's modulus for steel pinion, $E_1 = 200 \text{ GPa}$
" " " " CI gear, $E_2 = 100 \text{ GPa}$
 - ⑫ Assuming service factor, $C_s = 1.2$
- Let:
⑬ Face width $b = 10 \text{ m}$
i.e. $\phi = 10$

* Design procedure:

Let subscripts 1 and 2 are used for pinion and gear respectively

① T_1 = mean torque to be transmitted by pinion:

$$T_1 = \frac{P \times 60 \times 10^3}{2\pi N_1} = 381971.8634 \text{ N-mm}$$

② $[T_1]$ = design torque for pinion: $[T_1] = T_1 \times C_s$
 $= 458366.2361 \text{ N-mm}$

③ No. of teeth on gear: $z_2 = Gz_1 = 48$

④ Calculation of module:

↳ Finding the weaker wheel:

$$\text{Pinion: } ([\sigma_b]_1 y_1) = 105 \times \pi \times \left(0.124 - \frac{0.684}{z_1}\right) = 20.8017 \text{ MPa}$$

$$\text{Gear: } ([\sigma_b]_2 y_2) = 60 \times \pi \times \left[0.124 - \frac{0.684}{z_2}\right] = 20.6874 \text{ MPa}$$

$$\therefore ([\sigma_b]_2 y_2) < ([\sigma_b]_1 y_1)$$

\therefore Gear is a weaker wheel. Hence design will be based upon gear.

$$\text{i.e. } ([\sigma_b] y)_{w.G} = ([\sigma_b]_2 y_2) = 20.6874 \text{ MPa}$$

Hence, module should be calculated using the beam strength of gear

$$[F_s]_{w.G} = ([\sigma_b] y)_{w.G} \times b \times m$$

$$\frac{2T_1}{m z_1} \leq ([\sigma_b] y)_{w.G} \times 10 \text{ mm}^2$$

$$\frac{2 \times 458366.2361}{16 \times 20.6874 \times 10} \leq m^3$$

$$\therefore m \geq 6.52 \approx \boxed{m=7}$$

⑤ Dimensions of the gear train:

$$D_1 = mZ_1 = 112 \text{ mm} ; \quad D_2 = mZ_2 = 336 \text{ mm} ; \quad b = 10m = 70 \text{ mm}.$$

⑥ F_s = beam strength of weaker gear:

$$F_s = ([\sigma_b] Y)_{wG} b \cdot m = 10136.826 \text{ N} \quad \text{--- ①}$$

⑦ F_d = dynamic load:

$$F_t = \frac{2 [T_1]}{D_1} = 8185.1113 \text{ N}$$

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$$\text{Velocity Factor, } C_v = \frac{4.5}{4.5 + V} = 0.7189.$$

$$\therefore F_d = \frac{F_t}{C_v} = 11385.1113 \text{ N}.$$

$\therefore F_d > F_s \therefore$ design fails w.r.t. to the beam strength of gear.

⑧ Revising the module: (by equating ~~was~~ $F_d = F_s$).

$$\text{Using: } F_s = ([\sigma_b] Y)_{wG} \times 10 m^2 \quad (\text{eq ①})$$

$$11385.1113 = 20.6874 \times 10 \times m^2$$

$$m = 7.42$$

$$\boxed{m \approx 8}$$

⑨ Revising the dimensions of the gear train:

$$D_1 = mZ_1 = 128 \text{ mm} ; \quad D_2 = 384 \text{ mm} ; \quad b = 10m = 80 \text{ mm}.$$

⑩ Revised beam strength of weaker gear

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⑪ Revised dynamic load:

$$F_t = 7161.9724 \text{ N}$$

$$V_1 = \frac{\pi D_1 N_1}{60} = 2.0106 \text{ m/s}.$$

$$\text{Velocity factor ; } C_v = \frac{4.5}{4.5 + V} = 0.6912.$$

$$F_d = \frac{F_t}{C_v} = 10361.9724 \text{ N}.$$

$\therefore F_d < F_s$: design is safe.

⑫ Calculation of wear load: (always for pinion as pinion is weaker w.r.t. wear failure)

$$K = \frac{\sigma_{es}^2 \sin \phi}{1.4} \left[\frac{1}{E_1} + \frac{1}{E_2} \right] = 0.9650 \text{ MPa}$$

$$Q = \frac{2G}{G+1} = \frac{3}{2}$$

$$\therefore F_w = D_1 Q K b = 14834.688$$

$\therefore F_w > F_d \quad \therefore$ design is safe.

Design of shafts:

Case 1: Under variable load

(i) Soderberg equation :- (ductile materials)

$$\frac{\sigma_m}{\sigma_{yt}} + \frac{k_f \sigma_a}{\sigma_e} = \frac{1}{N} \quad (N_1 = N_2 = N)$$

(ii) Goodman's equation :- (Brittle material)

$$(k_t) \left[\frac{\sigma_m}{\sigma_{ut}} \right] + (k_f) \left[\frac{\sigma_a}{\sigma_e} \right] = \frac{1}{N} \quad (N_1 = N_2 = N)$$

where, $\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$ and $\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$;

* $k_f = 1 + q (k_t - 1)$

* $\sigma_e = \sigma_e^* K_a K_b K_c$ = endurance limit of a mechanical component (corrected E.L.).

where σ_e^* = E.L. of a std. specimen. → obtained from SN curve

= E.L. under reversed bending.

= 0.5 S_{ut} for Steels.

= 0.4 S_{ut} for CI.

K_a = size factor; K_b = S.F. factor.

K_c = load factor $\begin{cases} = 1 & \text{for reversed bending.} \\ = 0.7 & \text{for reversed axial load.} \\ = 0.6 & \text{for reversed torsion.} \end{cases}$

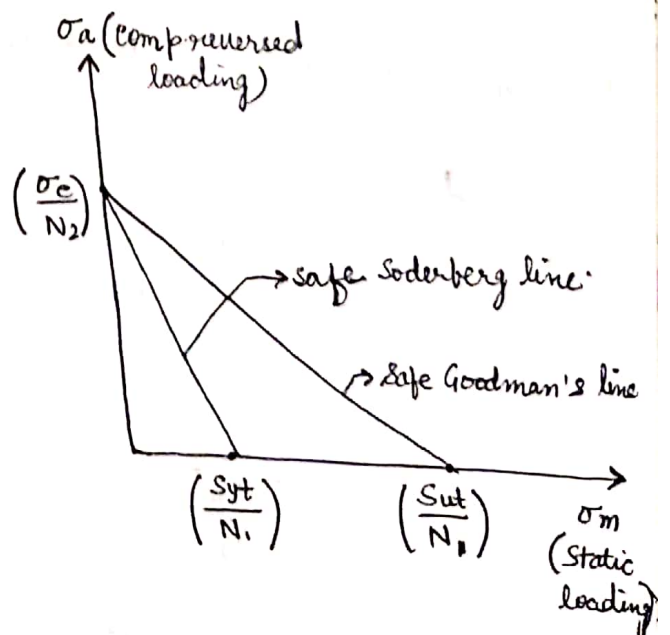
∴ Soderberg eqⁿ (Actual) :

↓
1 for ductile.

$$(k_t) \frac{\sigma_m}{\left(\frac{S_{yt}}{N_1} \right)} + (k_f) \frac{\sigma_a}{\left(\frac{\sigma_e}{N_2} \right)} = 1.$$

Goodman's eqⁿ (Actual) :

$$(k_t) \frac{\sigma_m}{\left(\frac{\sigma_{ut}}{N_1} \right)} + (k_f) \frac{\sigma_a}{\left(\frac{\sigma_e}{N_2} \right)} = 1$$



* Soderberg, Goodman and Gerber's eqⁿ are compulsory for design of component under alternating and fluctuating fatigue load. because failure stress of material is unknown when mean and variable stresses are non zero.

* Strength criterion, Soderberg, Goodman and Gerber's eqⁿ will give same result under completely reversed fatigue loading. [$\sigma_m = 0$, $\sigma_v = \sigma_{max}$] because failure stress of a material [i.e. σ_{es}] is known. Hence soderberg goodman and gerberg eqⁿ are optional under completely reversed fatigue loading.

* When $\sigma_m \neq 0$ all the three eqⁿ will give different results, hence best eqⁿ should be selected for safe design.

* Soderberg's equation:

↳ When variable A.L. or variable B.M alone:-

$$\frac{1}{N} = \frac{\sigma_m}{\sigma_{yt}} + k_f \frac{\sigma_a}{\sigma_e} \quad \text{--- (I)}$$

↳ When variable T.M acts alone:-

$$\frac{1}{N} = \frac{\tau_m}{\tau_{ys}} + k_f \frac{\tau_a}{\tau_e} \quad \text{--- (II)}$$

↳ When more than one variable load is acting;

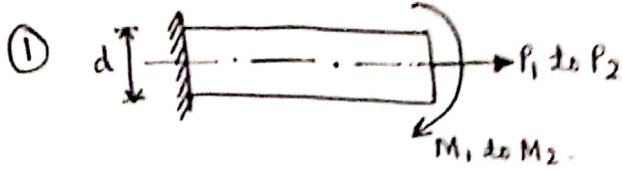
$$\frac{S_{yt}}{N} = \sigma_m + K_f \frac{\sigma_a S_{yt}}{\sigma_e} \quad \text{(from eq I)}$$

$$\therefore \boxed{\sigma_{eq} = \sigma_m + K_f \frac{\sigma_a \sigma_{yt}}{\sigma_e}} \quad \text{--- (III)}$$

For T.M :

$$\frac{S_{yso} \tau_{ys}}{N} = \tau_m + \frac{K_f \tau_a \tau_{ys}}{\tau_e}$$

$$\boxed{\tau_{eq} = \tau_m + \frac{K_f \tau_a \tau_{ys}}{\tau_e}} \quad \text{--- (IV)}$$



From eq. III:

$$(\sigma_{eq})_a = \frac{4 P_m}{\pi d^2} + k_f \frac{4 P_a}{\pi d^2} \cdot \frac{\sigma_{yt}}{\sigma_e}$$

$$(\sigma_{eq})_a = \frac{X}{d^2} \text{ MPa} \quad \text{--- (A)}$$

From eq. III:

$$(\sigma_{eq})_b = \frac{32 M_m}{\pi d^3} + k_f \left[\frac{32 M_a}{\pi d^3} \right] \left[\frac{\sigma_{yt}}{\sigma_e} \right]$$

$$(\sigma_{eq})_b = \frac{Y}{d^3} \quad \text{--- (B)}$$

$$\therefore (\sigma_{eq})_R = (\sigma_{eq})_a + (\sigma_{eq})_b = \frac{X}{d^2} + \frac{Y}{d^3} \quad \text{--- (C)}$$

$$(\sigma_{max})_{ind} \leq \sigma_{per}$$

$$\frac{X}{d^2} + \frac{Y}{d^3} \leq \frac{S_{yt}}{N}$$

$$d \geq \quad \text{mm}$$



From eq. III:

$$(\sigma_{eq})_a = \frac{4 P_m}{\pi d^2} + k_f \frac{4 P_a}{\pi d^2} \cdot \frac{\sigma_{yt}}{\sigma_e}$$

$$(\sigma_{eq})_a = \frac{X}{d^2} \text{ MPa} \quad \text{--- (A)}$$

From eq. IV:

$$\tau_{eq} = \frac{16 T_m}{\pi d^3} + k_f \frac{16 T_a}{\pi d^3} \left(\frac{\tau_{ys}}{\tau_e} \right)$$

$\frac{S_{yt}}{2}$ MSST
 $\frac{S_{yt}}{\sqrt{3}}$ MDET

$$\tau_{eq} = \frac{Z}{d^3} \text{ MPa} \quad \text{--- (B)}$$

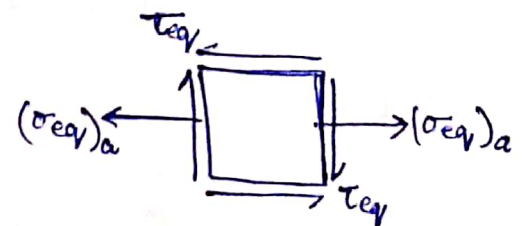
by using MSST:

$$(\sigma_t)_{per} = \sqrt{\sigma_x^2 + 4 \tau_{xy}^2}$$

$$\tau_{per} = \frac{1}{2} \sqrt{\sigma_x^2 + 4 \tau_{xy}^2}$$

$$\frac{S_{ys}}{N} = \frac{1}{2} \sqrt{(\sigma_{eq})_a^2 + 4 \tau_{eq}^2}$$

$$d = \quad \text{mm}$$



by using MDET:

$$(\sigma_t)_{\text{per}} = \sqrt{\sigma_x^2 + 3\tau_{xy}^2}$$

$$\tau_{\text{per}} = \frac{1}{\sqrt{3}} \sqrt{\sigma_x^2 + 3\tau_{xy}^2}$$

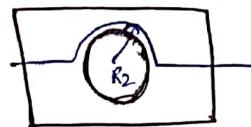
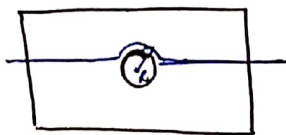
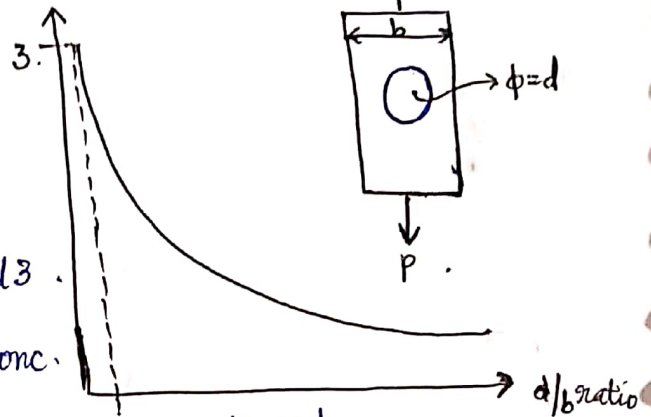
$$\frac{\sigma_{\text{ye}}}{N} = \frac{1}{\sqrt{3}} \sqrt{(\sigma_{\text{eq}})_a^2 + 3(\tau_{\text{eq}})^2}$$

* Stress concentration factor for circular hole:

* For circular hole,
max stress conc. factor = 3.
its actual value depends on $\frac{d}{b}$ ratio

Hence, stress conc. factor lies b/w 1 and 3.

* Smaller hole will have higher stress conc. factor.



$K_t \propto \frac{1}{R}$
 $R_1 < R_2$
Stress conc. in first case is more.

Q A hot rolled steel shaft subjected to torsional moment varies from 300 kN-mm (to CW) to 100 kN-mm (ACW) and bending moment at critical crosssection varies from 400 kN-mm to -200 kN-mm. The shaft is of uniform diameter and no keyway is present at the critical x-s/c. Determine diameter of shaft by taking FOS = 1.5. Assume ultimate, yield and design stress at 560 MPa, 420 MPa and 280 MPa. Modification factor = 0.62; size correction factor = 0.85. Load factor for bending = 1. Load factor for torsion = 0.58.

Soln. Given:

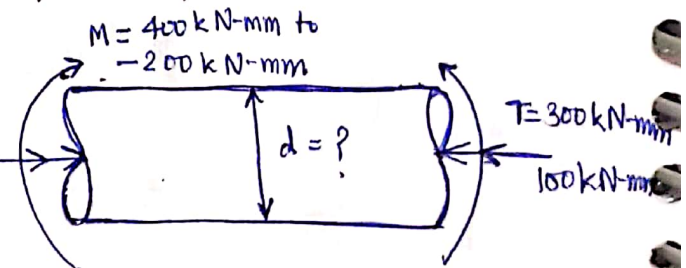
$K_t = K_f = 1$ [because of no discontinuity]

$S_{yt} = \sigma_{yt} = 420 \text{ MPa}$

$S_{ut} = \sigma_{ut} = 560 \text{ MPa}$

Design stress (σ_e^*) = 280 MPa.

Size factor, $K_a = 0.85$



Surface finish correction factor $= k_b = 0.62$.

Load factor for bending $K_c = 1$

Load factor for torsion $K_c = 0.58$

F.O.S. $N = 1.5$.

① Assuming variable bending moment acting alone:-

Soderberg eqⁿ is used because of ductile material (hot rolled steel).

$$M_{\max} = 400 \text{ kN-mm} ; M_{\min} = -200 \text{ kN-mm}.$$

$$M_m = \frac{M_{\max} + M_{\min}}{2} = 100 \text{ kN-mm} ; M_a = \frac{M_{\max} - M_{\min}}{2} = 300 \text{ kN-mm}.$$

$$\sigma_m = \frac{32 M_m}{\pi d^3} = \frac{320000}{\pi d^3} \text{ MPa} ; \sigma_a = \frac{32 M_a}{\pi d^3} = \frac{96 \times 10^5}{\pi d^3} \text{ MPa}.$$

$$\sigma_e = \sigma_e^* k_a k_b k_c = 280 \times 0.85 \times 0.62 \times 1 = 147.56 \text{ MPa}.$$

Hence, from Soderberg eqⁿ:

$$\sigma_{eq} = \sigma_m + K_f \frac{\sigma_a \sigma_{yt}}{\sigma_e} = \frac{30.5245 \times 10^6}{\pi d^3} \text{ MPa} \quad \text{--- ①}$$

② Assuming variable twisting moment alone:-

$$T_{\max} = 300 \text{ kN-mm} ; T_{\min} = -100 \text{ kN-mm}.$$

$$T_m = \frac{T_{\max} + T_{\min}}{2} = 100 \text{ kN-mm} ; T_a = \frac{T_{\max} - T_{\min}}{2} = 200 \text{ kN-mm}.$$

$$\tau_m = \frac{16 T_m}{\pi d^3} = \frac{16 \times 10^5}{\pi d^3} \text{ MPa} ; \tau_a = \frac{16 T_a}{\pi d^3} = \frac{32 \times 10^5}{\pi d^3} \text{ MPa}.$$

$$\tau_e = \sigma_e^* (k_a)(k_b)(k_c) = 280 \times 0.85 \times 0.62 \times 0.58 = 85.5848 \text{ MPa}.$$

Using Soderberg's eqⁿ:

$$\tau_{eq} = \tau_m + K_f \frac{\tau_a \tau_{ys}}{\tau_e}$$

[Using MSST; $\tau_{ys} = \frac{\sigma_{yt}}{2}$]

$$\tau_{eq} = \frac{9.4513 \times 10^6}{\pi d^3} \text{ MPa}$$

$$\tau_{ys} = \frac{420}{2}$$

③ Diameter of shaft: (d)

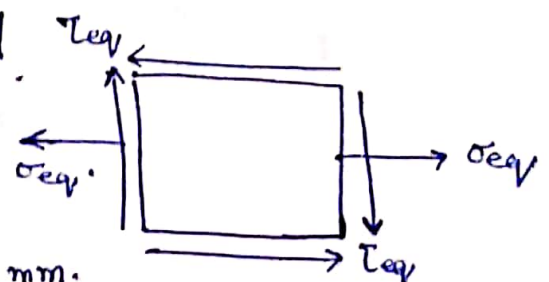
diameter is determined by using MSST because of biaxial state of stress and ductile material.

$$\tau_{per} = \sqrt{(\sigma_{eq})^2 + 4(\tau_{eq})^2}$$

$$\frac{420}{2 \times 1.5} = \frac{35.9040 \times 10^6}{\pi d^3 \times 2}$$

$$d = 34.433798 \text{ mm}.$$

$$d \approx 35 \text{ mm} \text{ [Standard dia in series of 5]}$$



Q For the stepped bar as shown determine for an infinite life, Assume $K_a = 0.9$, $K_b = 0.8$, $K_t = 1.5$, $q = 0.9$; $S_{yt} = 250 \text{ MPa}$, $S_{ut} = 400$, $F_{0.5} = 2$

$$\frac{[(\sigma_b)_{\max}]_B}{[(\sigma_b)_{\max}]_A} = \frac{(M_{\max})_B}{(M_{\max})_A} \cdot \left[\frac{d_A}{d_B} \right]^3$$

$$= \frac{1}{3} [8] = \frac{8}{3}$$

$$\frac{(\sigma_{\text{axial}})_B}{(\sigma_{\text{axial}})_A} = \left(\frac{d_A}{d_B} \right)^2 = 4$$

Hence, critical x-s/c is the x-s/c where dis continuity present. i.e. at B.

$$[\because (\sigma_b)_B = \frac{8}{3} (\sigma_b)_A \text{ and } (\sigma_a)_B = 4 (\sigma_a)_A]$$

* Stress analysis should be carried at x-s/c 'B'.

① When 'W' is acting alone:

$$\sigma_m = \frac{32 M_m}{\pi d^3} = \frac{32 \times 100 \times 2.25 \times 10^3}{\pi \left(\frac{d}{2} \right)^3} = \frac{57.6 \times 10^6}{\pi d^3}$$

$$\sigma_a = \frac{32 M_a}{\pi d^3} = \frac{32 \times 100 \times 1.75 \times 10^3}{\pi \left(\frac{d}{2} \right)^3} = \frac{44.8 \times 10^6}{\pi d^3}$$

$$K_f = 1 + q(K_t - 1) = 1.45$$

$$\sigma_e = (\sigma_e^*) (K_a \cdot K_b \cdot K_c) = 144 \text{ MPa}$$

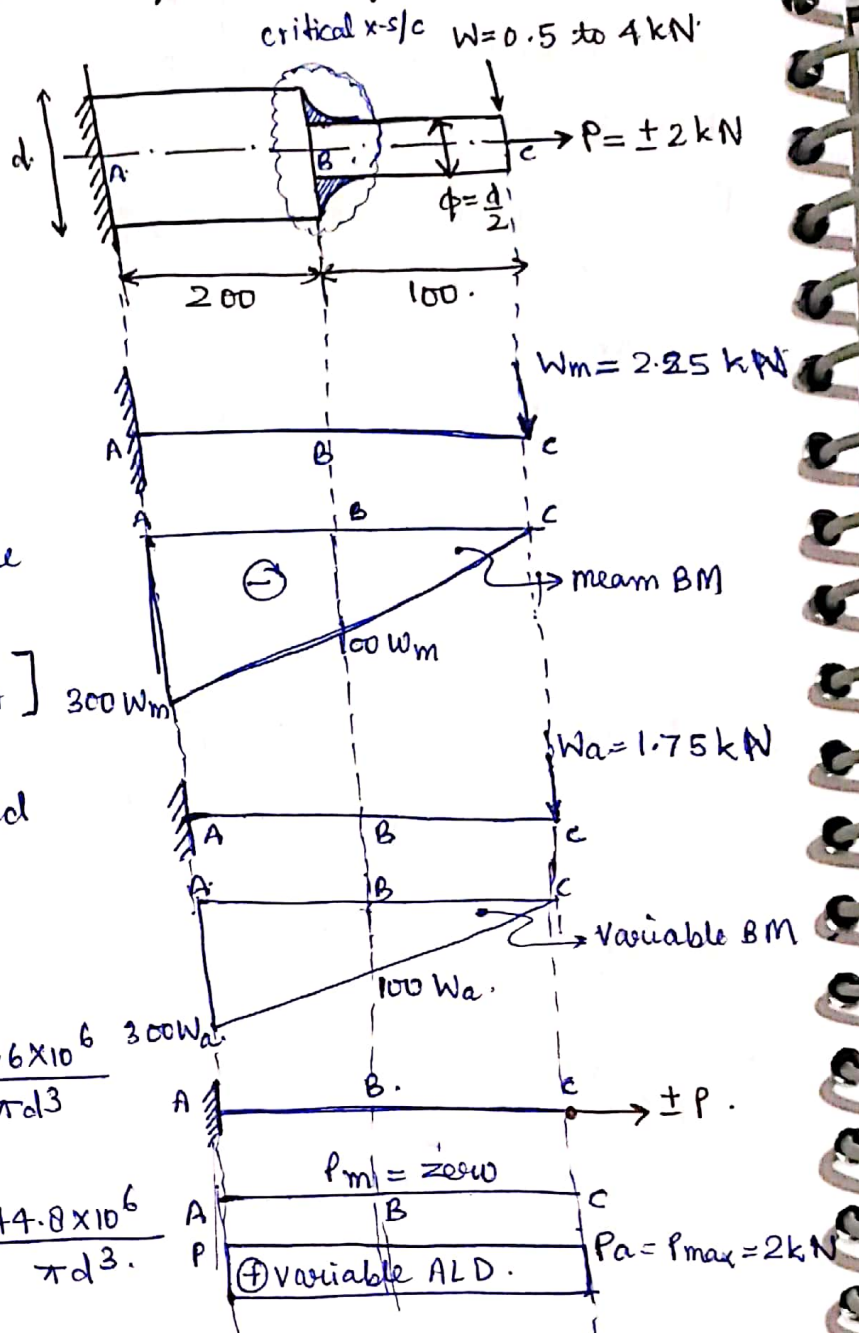
by using Soderberg's eqⁿ:

$$(\sigma_{eq})_B = \sigma_m + K_f \frac{\sigma_a \sigma_{yt}}{\sigma_e} = \frac{17.2629 \times 10^6}{\pi d^3} \text{ MPa}$$

When completely reversed axial load (P) acts:

$$\sigma_m = 0$$

$$\sigma_a = \sigma_{\max} = \frac{4 P_a}{\pi d_B^3} = \frac{324 \times 10^3}{\pi d^2}$$



$$\sigma_e^* = 50\% \text{ of } S_{ut}$$

$$= 200$$

$$(\sigma_{eq})_a = 0 + K_f \frac{\sigma_a \sigma_{st}}{\sigma_e} = \frac{11.6599 \times 10^3}{\pi d^2} \text{ MPa}$$

$$\sigma_e = \sigma_e^* K_a K_b K_c = 100.8 \text{ MPa}$$

$$K_c = 0.7$$

∴ diameter of bar (d):

$$(\sigma_{max})_{ind} \leq \sigma_{per}$$

$$\frac{17.2629 \times 10^6}{\pi d^3} + \frac{11.6599 \times 10^3}{\pi d^2} \leq \frac{250}{2}$$

$$d \geq 76.994 \text{ mm}$$

$$d = 77 \text{ mm}$$

$$\sigma_x \leftarrow \boxed{} \rightarrow \sigma_x = (\sigma_{eq})_b + (\sigma_{eq})_a$$

Fig. State of stress at critical plane (i.e. top fibre of critical x-s/c (B)).

Q The non-rotating shaft is subjected to load P varying 4 kN to 12 kN.

$S_{ut} = 600 \text{ MPa}$; $S_{yt} = 350 \text{ MPa}$; Endurance limit = 300 MPa;

size factor = 0.8, surface finish factor = 0.85, $q = 0.9$;

FOS = 3.5, $K_t = 1.8$. Find the diameter D using Goodman's eqⁿ:

$$\frac{(\sigma_b)_{B+D}}{(\sigma_b)_C} = \left[\frac{M_B}{M_C} \right] \left[\frac{d_C}{d_B} \right]^3$$

mean σ_b :-

$$\frac{(\sigma_b)_{B+D}}{(\sigma_b)_C} = \frac{240}{240} \times 2^3 = 4.8$$

Variable σ_b :-

$$\frac{(\sigma_b)_{B+D}}{(\sigma_b)_C} = \frac{120}{120} \times 2^3 = 4.8$$

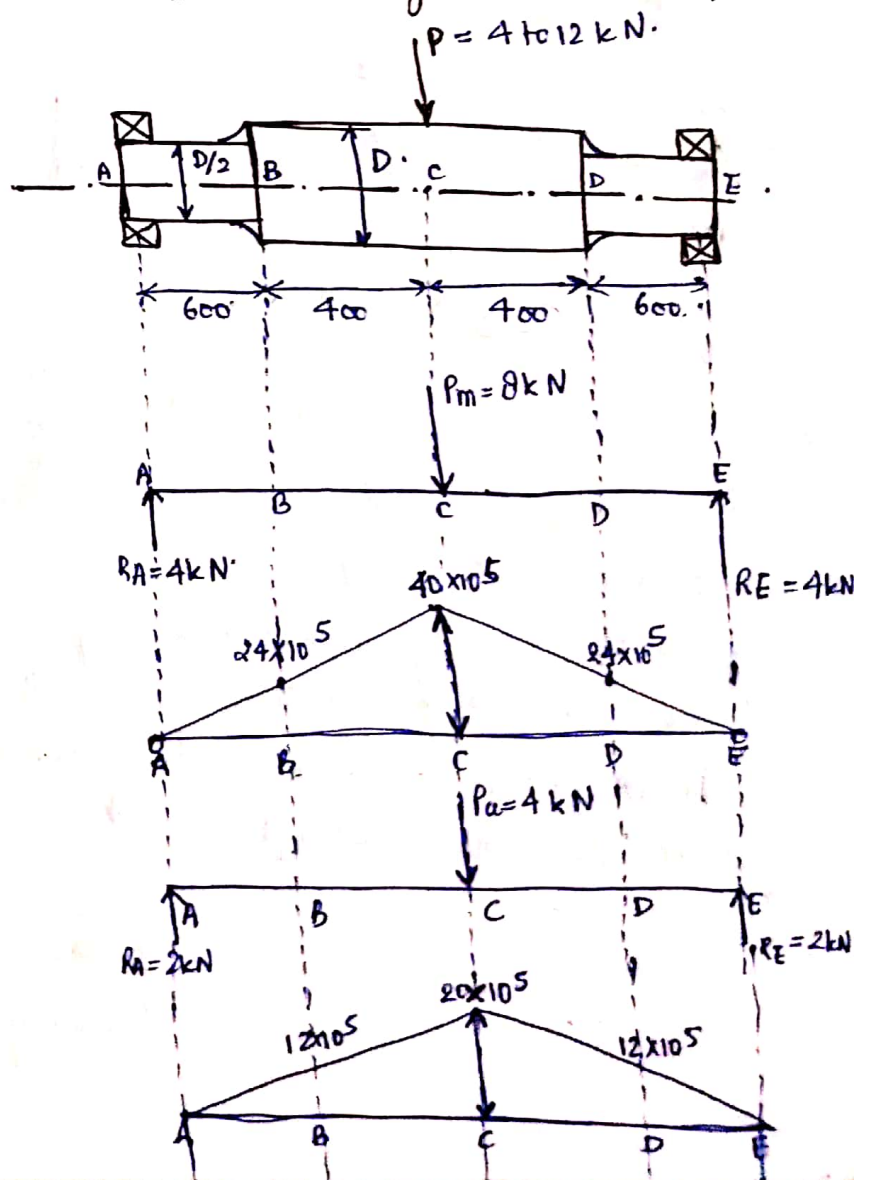
At critical x-s/c;

$$\sigma_m = \frac{32 M_m}{\pi d^3} = \frac{32 \times 24 \times 10^5}{\pi D^3} \times 8$$

$$= \frac{614.4 \times 10^6}{\pi D^3}$$

$$\sigma_v = \frac{32 M_a}{\pi d^3} = \frac{32 \times 12 \times 10^5}{\pi D^3} \times 8$$

$$= \frac{307.2 \times 10^6}{\pi D^3}$$



$$K_f = 1 + q(k_t - 1) = 1.72.$$

$$\sigma_e = (\sigma_e^*) (k_a)(k_b)(k_c) = 204 \text{ MPa}.$$

Using Goodman's eqⁿ:

$$\frac{1}{N} = K_t \left(\frac{\sigma_m}{\sigma_{ut}} \right) + K_f \left(\frac{\sigma_a}{\sigma_e} \right).$$

$$\frac{1.8432 \times 10^6}{\pi d^3} + \frac{2.5901 \times 10^6}{\pi d^3} = \frac{1}{3.5}$$

$$d = 170.3 \text{ mm}$$

$$d \approx 175 \text{ mm}.$$

Q For the rotating shaft as shown in fig. Determine the life of shaft.
 $S_{ut} = 500 \text{ MPa}$; endurance limit of shaft is 200 MPa ,
 $K_f = \text{fatigue stress conc. factor} = 2.$

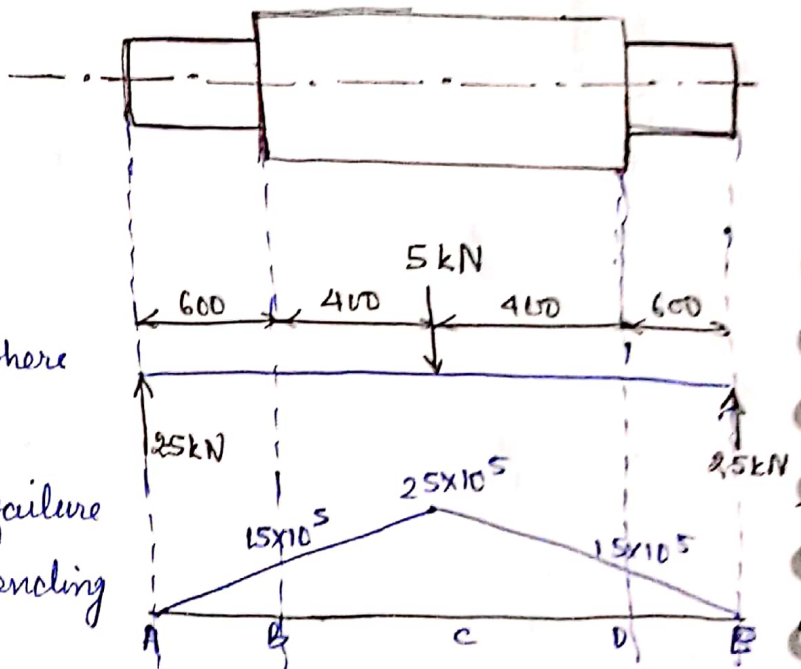
$$\frac{(\sigma_b)_{B\&D}}{(\sigma_b)_C} = \frac{M_B}{M_C} \times \left(\frac{d_C}{d_B} \right)^3$$

$$\frac{(\sigma_b)_{B\&D}}{(\sigma_b)_C} = \frac{15}{25} \times 2^3 = 4.8$$

* Critical x-s/c are B & D (i.e. where fillets are provided).

* Shaft is subjected to fatigue failure because completely reversed bending stresses are developed at the

critical points (i.e. on bottom fibres) on the critical x-s/c due to rotation of shaft.



fatigue loading.
 failure is under tension only.
 (rarely in compression)

Critical cross-section:

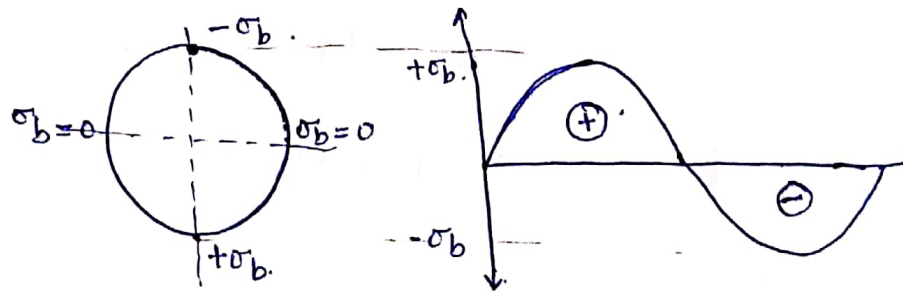


Fig. Stress cycle for comp. reversed bending stress.

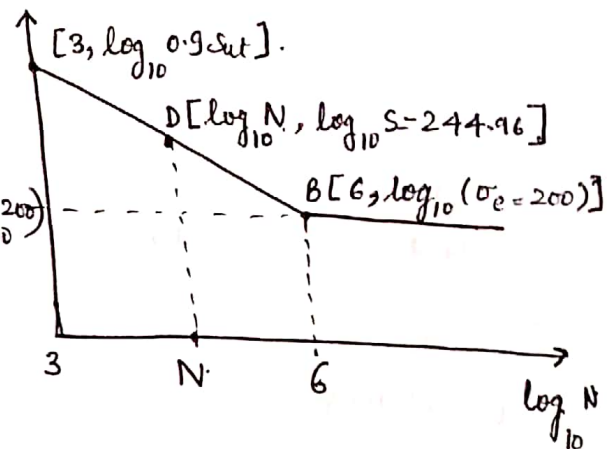
$$(\sigma_{\max}) = K_f (\sigma_b)_{\max} = \frac{2 \times 32 M}{\pi (50)^3} = 244.96 \text{ MPa} > \sigma_e.$$

Hence, shaft will have finite life.

$$\frac{\log_{10}(0.9 \times 500) - \log_{10}(200)}{\log_{10}(244.46) - \log_{10}(200)} = \frac{6 - 3}{6 - \log_{10} N}$$

$$N = 180868.1 \text{ cycles.}$$

$$N = 180869 \text{ cycles.}$$



* If diameter is 75 mm instead of 50 mm Shaft will have infinite life because, the max stress is less than σ_e .

$$\text{i.e. } \sigma_{\max} = 72.43 < 200 \text{ MPa.}$$

Q A m/c component is subjected to biaxial state of stress as shown. Determine FOS by using MDET. Assume, $S_{yt} = 250 \text{ MPa}$, $S_{ut} = 500 \text{ MPa}$.

$K_t = 1.85$; $q = 0.9$; Corrected endurance limit = 200 MPa.

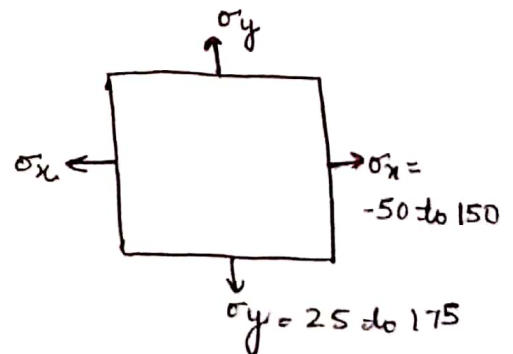
$$(\sigma_x)_m = \frac{150 - 50}{2} = 50 \text{ MPa.}$$

$$(\sigma_x)_a = \frac{150 + 50}{2} = 100 \text{ MPa.}$$

$$K_f = 1 + q(K_t - 1) = 1.765.$$

$$\sigma_e = \sigma_e^* K_a K_b K_c = 200 \text{ MPa.}$$

$$(\sigma_x)_{eq} = \sigma_m + K_f \frac{\sigma_a \sigma_{yt}}{\sigma_e} = 270.625 \text{ MPa.}$$



$$(\sigma_y)_{\text{mean}} = 100 \text{ MPa}; \quad (\sigma_y)_a = 75 \text{ MPa}.$$

$$(\sigma_y)_{eq} = 100 + 1.765 \times \frac{75 \times 250}{200} = 265.46875 \text{ MPa}.$$

$$\sigma_1 = 270.625 \text{ [larger of } (\sigma_x)_{eq} \text{ \& } (\sigma_y)_{eq} \text{]}.$$

$$\sigma_2 = 265.46875 \text{ MPa}.$$

by using MDET.

$$\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2 \leq \left(\frac{S_{yt}}{FOS} \right)^2$$

$$FOS = 0.9325$$

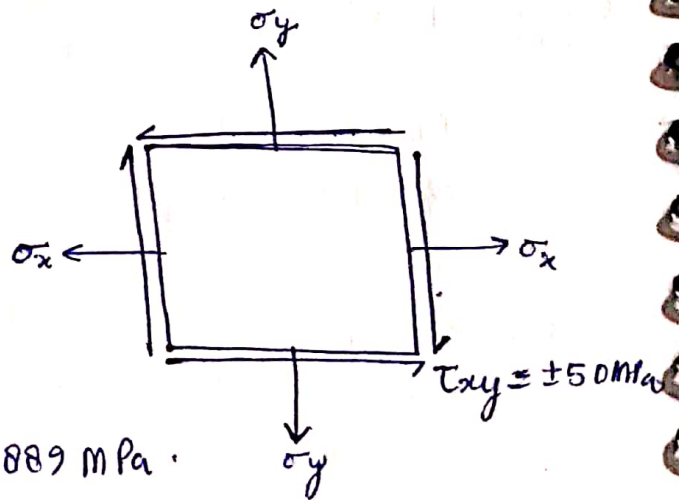
Q If τ_{xy} is also given, .

$$\tau_m = 0$$

$$\tau_a = 50 \text{ MPa}.$$

$$\tau_{eq} = \tau_m + K_f \frac{\tau_a \tau_{sy}}{\tau_e \rightarrow \sigma_c \times K_c}$$

$$= 0 + \frac{1.765 \times 50 \times 250}{200 \times \sqrt{3}} = 63.6889 \text{ MPa}.$$



Q $S_{ut} = 400 \text{ MPa}$.

$S_{yt} = 250 \text{ MPa}$.

$K_f = 2$; $N = 1.5$; $K_a = 0.85$; $K_b = 0.7$

